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Cosmic strings and galaxy formation

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The role of strings in the early Universe is reviewed, with particular emphasis on the possibility that they may provide the fluctuations that lead to galaxy formation. Evidence from the correlation of clusters is presented. An alternative scenario, in which much lighter strings might come to dominate the recent Universe is also discussed briefly.

1. INTRODUCTION

There has been considerable progress recently in understanding the role that cosmic strings could play in cosmology, especially in seeding galaxy formation. Several of the most interesting theories of fundamental particle interactions are now known to predict such strings. Through a combination of numerical and analytical work the processes of formation and evolution of a network of cosmic strings are now much better understood.

In this talk we review these ideas, with emphasis on two recent developments: the remarkable agreement between the spatial correlation function for string loops and that for Abell clusters, which adds weight to the string theory of galaxy formation, and the alternative suggestion that our Universe may be dominated by lighter strings.

We begin by reviewing the conditions under which strings are formed and their initial configuration. The core of the discussion concerns the subsequent evolution of the system of strings. This evolution leads to one of two possible end results: either a string-dominated Universe or a 'scaling' solution in which the scale size of the string system is a constant fraction of the horizon size. Then we discuss the cosmological implications of the scaling solution, especially the role of loops as seeds for galaxy formation and the evidence concerning the spatial correlation functions. We conclude by discussing other possible observable effects.

2. THEORIES THAT PREDICT STRINGS

Strings may be formed at a phase transition in the early universe at which a gauge symmetry group G is broken to a subgroup H. Typically this happens because a Higgs field Φ acquires a non-zero value, say Φ_0. Then H is the subgroup of G leaving Φ_0 unchanged. The possible values of Φ_0 are the points of the surface M of minima of some potential function. These points are in one-to-one correspondence with left cosets of H in G, i.e. M may be identified with the quotient space G/H.

The necessary and sufficient condition for the existence of strings is that M be non-simply connected, i.e. contain non-contractible loops or, equivalently, that the first homotopy group of M, namely $\pi_1(G/H)$, be non-trivial.

There are two main classes of strings. The more familiar arise from the complete breaking of a U(1) symmetry. In this case the first homotopy group is Z, so the strings are labelled by an integer-valued quantum number. Examples of such strings are vortex lines in superfluids and flux tubes in superconductors.

On the other hand, strings labelled by a finite group arise in the breaking of a simply connected group G to a subgroup H with disconnected parts. In fact, $\pi_1(G/H)$ may be identified with the zeroth homotopy group $\pi_0(H)$, the quotient of H by its connected subgroup. Many grand unified theories of fundamental particle interactions exhibit this phenomenon. Two examples are the symmetry breaking schemes

$$\mathrm{SO}(10) \rightarrow \mathrm{SU}(5) \times Z_2$$

and

$$E_6 \rightarrow [\mathrm{SO}(10) \times Z_8]/Z_4,$$

both of which yield Z_2 strings (Kibble *et al.* 1982; Olive & Turok 1982).

The currently very popular superstring theories also yield cosmic strings. In the SO(32) superstring theory the centre $Z_2 \times Z_2$ of the universal covering group of SO(32) remains unbroken, giving for example the breaking scheme

$$\mathrm{SO}(32) \rightarrow \mathrm{SU}(5) \times Z_2 \times Z_2.$$

In the $E_8 \times E_8$ superstring theory, most of the favoured symmetry-breaking patterns produce extra U(1) factors, which yield strings with integer quantum numbers when they break (Witten 1985).

Most theories that predict strings are characterized by a single relevant mass scale m. Both the critical temperature T_c and the magnitude $|\Phi|$ of the zero-temperature Higgs field are roughly of order m. The string tension, or mass per unit length, μ is of order πm^2. A particularly important parameter is the dimensionless quantity $G\mu$, where G is Newton's constant. For example, for m in the range 10^{15}–10^{16} GeV, $G\mu$ lies between 10^{-7} and 10^{-5}.

3. Initial configuration

Shortly after the phase transition at which strings are formed we expect a random, or 'Brownian', configuration of strings with a persistence length ξ related to the correlation length of the Higgs field. This may be of order m^{-1}, and is certainly small compared with the horizon size at the time. We expect a length of approximately ξ of string in each volume ξ^3, so that their contribution to the mass density is

$$\rho_s \approx \mu/\xi^2. \tag{1}$$

Vachaspati & Vilenkin (1984) have performed a numerical simulation of U(1) strings on a cubic lattice, by assigning a random phase to the Higgs field at each lattice site. They find that 80% of the total length of string is in the form of 'infinite' strings, i.e. strings long compared with the size of the lattice. The remaining 20% is in the form of loops whose distribution follows a scaling law; the number density of loops of size r (where r is, say, the RMS radius) is

$$n(r)\,\mathrm{d}r \propto (1/r^3)\,\mathrm{d}r/r. \tag{2}$$

Moreover, the perimeter $l(r)$ of the loop is proportional, as one would expect for a random walk, to r^2.

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One of us (Kibble 1986*a*) has recently shown that similar results obtain for the Z_2 strings arising in the complete breaking of SO(3) symmetry, with the interesting difference that less than 10% of the length is in the form of loops.

4. Evolution of strings

In the early stages of evolution following their formation the strings are heavily damped by interaction with the dense surrounding medium. During this time, the total length of string decreases and the persistence length ξ increases. This process continues until ξ is of the same order as the horizon size, which occurs when the temperature is $T_* \approx m^2/m_{\mathrm{Pl}}$, where m_{Pl} is the Planck mass (Kibble 1976, 1982; Everett 1981).

The subsequent evolution depends critically on what happens when strings intersect. There are two possibilities: either they pass through one another more or less unaltered or they exchange partners. Let us call the probability of exchanging partners (the 'intercommuting probability') p.

The only direct evidence we have concerning p comes from numerical simulations by P. Shellard (unpublished results 1985). These show that p is close to unity except when the relative velocity of the strings when they meet is very large, in excess of $0.9c$. It therefore seems reasonable to assume that p is usually large.

The opposite extreme, $p = 0$, has been discussed by Vilenkin (1984*a*). He showed that in that case the string density ρ_s would fall only as $t^{-\frac{4}{3}}$, so that eventually strings must come to dominate. We shall assume however that p is not close to zero.

A crucial role in string evolution is played by the process of formation of closed loops by self-intersection, and their subsequent decay by gravitational radiation (Vilenkin 1981*a*). This is the principal means by which the system of strings loses energy.

Recently, Albrecht & Turok (1985) have performed a numerical simulation of the evolution of strings with $p = 1$. They find that the strings evolve rapidly towards a scaling solution in which the persistence length $\xi \propto t$. This means that $\rho_s \approx \mu/t^2$, so the strings form a fixed fraction of the total density,

$$\rho_s/\rho \approx G\mu \approx 10^{-6}. \tag{3}$$

However, in this case the loops contribute even more; if ρ_l is the loop density,

$$\rho_l/\rho \approx (G\mu)^{\frac{1}{2}} \approx 10^{-3}. \tag{4}$$

It is possible with various assumptions to analyse the evolution of the system of strings (Kibble 1985, 1986*b*). Denoting the ratio ξ/t by γ, one finds an equation of the form

$$t\dot{\gamma}/\gamma = f(\gamma).$$

The function $f(\gamma)$ is negative at $\gamma = 0$ and at $\gamma \rightarrow \infty$ and rises to a maximum between. If this maximum is positive, there will be a stable scaling solution, corresponding to the larger root of $f(\gamma) = 0$. On the other hand, if the maximum is negative, no scaling solution exists and γ will evolve towards $\gamma = 0$, leading to eventual string domination. It is hard to know *a priori* which of these alternative outcomes to expect. This requires a more thorough study (but see Bennett 1985).

Observationally we know that strings could not have dominated the Universe until at least

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$t \approx 10^4$ years, otherwise they would wreck the nucleosynthesis scenario. String domination would be approached rather slowly

$$\xi/t \propto t^{-k}, \quad \tfrac{1}{6} < k < \tfrac{1}{4}.$$

Thus one finds (Kibble 1986*b*) that string domination is consistent only for a very small value of μ,

$$G\mu \approx 10^{-14.5}\text{–}10^{-12}$$

corresponding to a critical temperature

$$T_c \approx 10^{12} - 10^{13}\ \text{GeV},$$

or

$$T_* \approx 10^{4.5}\text{–}10^{7}\ \text{GeV}.$$

If μ is in this range, then it is possible that strings provide the dark matter required to make $\Omega = 1$. However, they certainly could *not* explain the dark matter in galactic halos required, according to some, by observations of rotation curves.

5. Loops and density perturbations in the scaling solution

Although a string-dominated Universe is conceivable, a more natural and attractive scenario is given by the scaling solution.

The picture that emerges from the numerical simulations of Albrecht & Turok (1985) is the following. The strings straighten on a scale *ca.* t (similar to the horizon size) and generate loops with a typical radius *ca.* t. These original parent loops each give rise by self-intersection to about ten daughter loops, which then survive to decay slowly by gravitational radiation.

Loops of size r are born when $r \approx t$, and have a typical separation also of order r. At a later time t, the separation has grown with the Universal expansion by a factor $(t/r)^{\frac{1}{2}}$. Thus in place of (2) the number density of loops is given by

$$n(r)\,\mathrm{d}r \approx (1/(rt)^{\frac{3}{2}})\,\mathrm{d}r/r. \tag{5}$$

Clearly the total mass density of loops is dominated by the smallest surviving loops, with $r/t \approx G\mu$.

Because of the way they are formed the positions of loops are strongly correlated. This correlation will show up in structures which have form around them, with the loops as seeds.

Consider a large volume of radius R. On average the number of loops it contains of size r will be

$$N \approx R^3/(rt)^{\frac{3}{2}}.$$

Hence the expected fluctuation in the mass contained in this volume is, for $N \gg 1$,

$$\delta M/M \approx N^{\frac{1}{2}}\,\mu r/\rho_b R^3.$$

Because $\delta M/M \propto r^{\frac{1}{4}}$, the mass fluctuation will be dominated by the largest loops for which $N \gtrsim 1$, with

$$r \approx R^2/t < R,$$

leading to a typical value (Turok 1984)

$$\delta M/M \approx \mu/(\rho_B\, tR).$$

However, there will be occasional fluctuations much larger than this 'typical' value, in the very rare cases when the volume R^3 contains a single loop of size *ca.* R. This gives

$$\delta M/M \approx \mu/(\rho_b R^2).$$

If we assume that these rare, very large fluctuations form the seeds for correspondingly large condensations, then we should expect the distribution of large loops to be reflected in the distribution of rich clusters of galaxies.

6. Correlation of loops and clusters

One of us (Turok 1985) has compared the loop–loop correlation function with the observed cluster–cluster correlation function ξ_{cc} of Abell clusters (Bahcall & Soneira 1983). These are defined as regions with more than 50 bright galaxies within $1.5\,h^{-1}$ Mpc†, compared with the mean separation of galaxies of about $5\,h^{-1}$ Mpc. Because the process of loop formation is essentially independent of scale, the loop–loop correlation function for loops of size r should be a function only of the dimensionless ratio r/d, where d is the mean separation of loops of this size. This function $\xi(r/d)$ was determined from simulations similar to those of Albrecht & Turok (1985).

Choosing d to be the mean separation of Abell clusters, namely $d_c \approx 55\,h^{-1}$ Mpc, we may then compare ξ with the observed cluster–cluster correlation function. This comparison is shown in figure 1 which is taken from Turok (1985). The agreement is remarkable, especially in view of the fact that there are no adjustable parameters in the theory except the intercommuting probability p which is here set equal to unity.

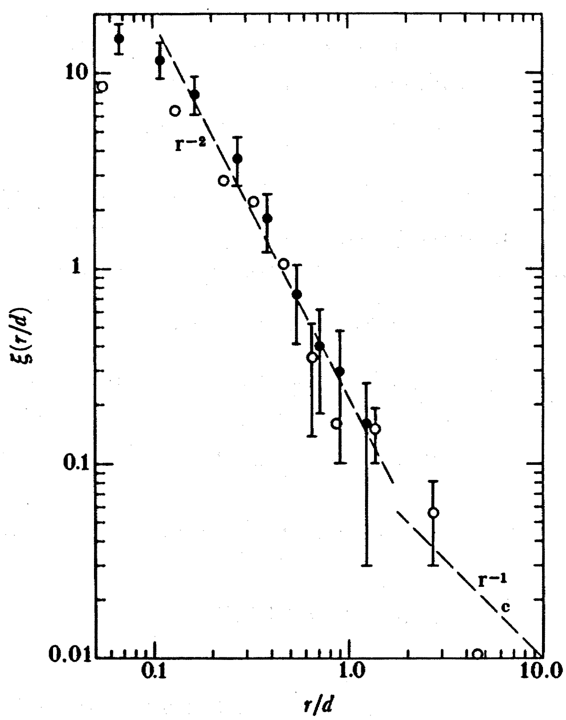

Figure 1. Comparison of ξ with the observed cluster–cluster correlation function.

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The correlation function $\xi(r/d)$ is independent of the value of the parameter $G\mu$. However, $G\mu$ can be estimated from the requirement that loops with mean separation d are sufficiently massive to virialize the mass excess typical of an Abell cluster, namely $\delta M/M \approx 170$, by the present epoch. This requirement yields (Turok & Brandenberger 1985)

$$G\mu \approx 2 \times 10^{-6} \tag{6}$$

(assuming that $\Omega = 1$ and $h = 0.5$).

The correlation function of galaxies yields an independent check of this estimate, by requiring that loops with the mean separation of galaxies give rise to objects as correlated as galaxies. Because $d_g \approx \frac{1}{10} d_c$ the primordial correlation function for the corresponding 'galaxy loops' is roughly one hundredth of that for the 'cluster loops'. A gravitational enhancement of this correlation function by a factor of about five is required to fit the observed galaxy–galaxy correlation function. A simple model for this gravitational enhancement (Turok & Brandenberger 1985) may be used to calculate $G\mu$. The result,

$$G\mu \approx 4 \times 10^{-6},$$

is in quite good agreement with (6) above, certainly well within the uncertainties in the calculation. This is encouraging.

7. Detection of strings

Finally, let us turn to the possibility of detecting cosmic strings themselves.

Their most direct observable effect is gravitational lensing. A straight string produces double images with a typical angular separation (Vilenkin 1981*b*, 1984*b*; Hogan & Narayan 1985)

$$\Delta\theta \approx 4\pi G\mu \approx 5''$$

for the value (6). All five of the known cases of gravitational lensing are in roughly the right range and only in one case has a candidate object to produce the lensing been seen. However, because many other types of objects can act as gravitational lenses, this can never be conclusive.

A much more definite prediction is of a specific type of anisotropy in the microwave background radiation. A string moving with velocity $v_\perp$ perpendicular to the line of sight produces a sharp discontinuity in the observed temperature (Kaiser & Stebbins 1984)

$$\delta T/T = 8\pi G\mu v_\perp \approx 2 \times 10^{-5}$$

with $v_\perp \approx 0.4$, the typical value for oscillating loops, which dominate the effect. This is not very far below the present observational limits. Detection of this effect would be a very definitive test of the cosmic string theory; it is very difficult to imagine any other mechanism that would yield anything similar.

The loops also give rise in the usual way to a Sachs–Wolfe effect (Brandenberger & Turok 1985), with

$$(\delta T/T)_{\mathrm{RMS}} \approx 5 \times 10^{-5} (\sin \tfrac{1}{2}\theta)^{\frac{1}{2}},$$

but this is not particularly distinctive.

Another important source of observable effects is the gravitational radiation emitted by decaying loops. In particular, Hogan & Rees (1984) have shown that this would lead to variations in the frequency of the millisecond pulsar. The fact that no such variations have been seen places an upper limit on the value of $G\mu$, namely

$$G\mu \lesssim 10^{-5}(\tfrac{\alpha}{T})^8,$$

where α is of order 2π and T is the time in years over which measurements have been made. Failure to see such variations within about a decade would serve to rule out the theory with the value (6).

Gravitational radiation may have other observable effects. In particular, requiring that the nucleosynthesis scenario be unaffected imposes an upper limit on $G\mu$, which, according to the parameters so far determined in numerical simulations, is consistent with (6) (different values were used by Bennett (1985) who found an inconsistency).

8. Conclusions

Strings appear in many models of fundamental particle interactions. If cosmic strings do appear, their subsequent evolution will lead either to a string-dominated Universe or to a scaling solution. String domination is possible only in the very recent Universe, which would imply that the strings are light ($10^{-14.5} \lesssim G\mu \lesssim 10^{-12}$).

In the scaling solution the loops play a particularly important role. They yield a very attractive theory of galaxy formation. In particular the correlation function of loops reproduces very well that of Abell clusters. This theory requires that $G\mu \approx 10^{-6}$.

Observable effects include gravitational lensing, with a typical angular separation between the images of 5″. The most definitive test would be the finding of predicted discontinuities in the observed temperature of the microwave background. There may also be important effects of gravitational radiation.

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